A note on hamiltonicity of generalized net-free graphs of large diameter

Jan Brousek *
Department of Mathematics
University of West Bohemia
306 14 Pilsen
Czech Republic
e-mail brousek@kma.zcu.cz

Ralph J. Faudree
Department of Math. Sciences
The University of Memphis
Memphis, TN 38152
U.S.A.
e-mail rfaudree@cc.memphis.edu

Zdeněk Ryjáček*
Department of Mathematics
University of West Bohemia
306 14 Pilsen
Czech Republic
e-mail ryjacek@kma.zcu.cz

September 23, 2000

Abstract

A generalized (i, j, k)-net $N_{i,j,k}$ is the graph obtained by identifying each of the vertices of a triangle with an endvertex of one of three vertex-disjoint paths of lengths i, j, k. We prove that every 2-connected claw-free $N_{1,2,j}$-free graph of diameter at least $\max\{7, 2j\}$ $(j \geq 2)$ is hamiltonian.

Keywords: hamiltonian graphs, forbidden subgraphs, claw-free graphs

1991 Mathematics Subject Classification: 05C45

*Research supported by grant GA ČR No. 201/97/0407

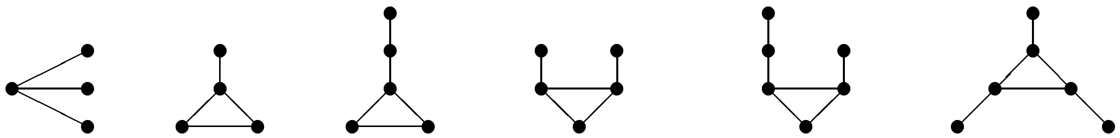
1 Introduction

In this paper we consider finite simple undirected graphs $G = (V(G), E(G))$ and for concepts and notations not defined here we refer the reader to [2].

For a set $S \subset V(G)$ we denote by $N(S)$ the *neighborhood* of S, i.e. the set of all vertices of G which have a neighbor in S. If $S = \{x\}$, we simply write $N(x)$ for $N(\{x\})$. For any subset $M \subset V(G)$, we denote $N_M(S) = N(S) \cap M$. If H is a subgraph of G, we write $N_H(S)$ for $N_{V(H)}(S)$. For subsets $M, N \subset V(G)$, $M \cap N = \emptyset$, we denote $E(M,N) = \{xy \in E(G) | \ x \in M, y \in N\}$. The induced subgraph on a set $M \subset V(G)$ in G will be denoted by $\langle M \rangle_G$.

We denote by $\mathrm{diam}(G)$ the *diameter* of G, i.e. the largest distance of a pair of vertices $x, y \in V(G)$. A path with endvertices x, y will be sometimes referred to as an *xy-path.* If x, z are vertices at distance $\mathrm{diam}(G)$, then any shortest xz-path will be called a *diameter path* of G. By $c(G)$ we denote the *circumference* of G, i.e. the length of a longest cycle in G. A graph G is *hamiltonian* if $c(G) = |V(G)|$.

If $H_1, \ldots, H_k$ ($k \geq 1$) are graphs, then a graph G is said to be *$H_1H_2\ldots H_k$-free* if G contains no copy of any of the graphs $H_1, \ldots, H_k$ as an induced subgraph. The graphs $H_1, \ldots, H_k$ will be also referred to in this context as *forbidden subgraphs.* Specifically, the graph $K_{1,3}$ will be also denoted by C and called the *claw* and in this case we say that G is *claw-free.* Whenever vertices of an induced claw are listed, its *center*, (i.e. its unique vertex of degree 3) is always the first vertex of the list. We denote by P_i the path on i vertices. Further graphs that will be often considered as forbidden subgraphs are shown in Figure 1.



The claw C $\quad Z_1 \quad Z_2 \quad$ The bull B $\quad$ The wounded W $\quad$ The net N

Figure 1

We say that a vertex $x \in V(G)$ is *locally connected* if $\langle N(x) \rangle_G$ is a connected graph. A locally connected vertex with a noncomplete neighborhood is called *eligible* and the graph G'_x, obtained from G by adding to the neighborhood of an eligible vertex x all missing edges, (i.e. such that $N(x)$ induces in G'_x a complete graph), is called the *local completion of G at x.* The following was proved in [12].

Theorem A [12]. *Let G be a claw-free graph and let x be an eligible vertex of G. Let $N'_x = \{uv \mid u, v \in N_G(x), uv \notin E(G)\}$ and let G'_x be the graph with vertex set $V(G'_x) = V(G)$ and with edge set $E(G'_x) = E(G) \cup N'_x$. Then*

(*i*) *the graph G'_x is claw-free,*

(*ii*) $c(G'_x) = c(G)$.

It can be shown that a graph which is obtained from a claw-free graph G by recursively performing the local completion operation, as long as there is at least one eligible vertex, is uniquely determined and is again claw-free. This graph is called the *closure of* G and is denoted by $\mathrm{cl}(G)$. The following theorem summarizes the basic properties of the closure.

Theorem B [12]. *Let G be a claw-free graph. Then*
(*i*) $\mathrm{cl}(G)$ *is well-defined (i.e., uniquely determined by G),*
(*ii*) *there is a triangle-free graph H such that* $\mathrm{cl}(G)$ *is the line graph of H,*
(*iii*) $c(\mathrm{cl}(G)) = c(G)$,
(*iv*) *G is hamiltonian if and only if* $\mathrm{cl}(G)$ *is hamiltonian.*

If $G = \mathrm{cl}(G)$, then we say that the graph G is *closed* (thus, G is closed if and only if G is the line graph of a triangle-free graph).

There are many results dealing with hamiltonian properties in classes of graphs defined in terms of forbidden induced subgraphs (see e.g. [10], [7], [11], [3], [4]). Bedrossian [1] (see also [8]) characterized all pairs X, Y of connected forbidden subgraphs implying hamiltonicity.

Theorem C [1]. *Let X and Y be connected graphs with X, $Y \neq P_3$, and let G be a 2-connected graph that is not a cycle. Then, G being XY-free implies G is hamiltonian if and only if (up to symmetry) $X = C$ and $Y = P_4, P_5, P_6, C_3, Z_1, Z_2, B, N$ or W.*

The classes of CB-free graphs and CN-free graphs were extended in [5] as follows. We denote by (see also Figure 2):

$B_{i,j}$ $(j \geq i \geq 1)$ – the *generalized* (i,j)*-bull*, i.e. the graph which is obtained by identifying each of some two distinct vertices of a triangle with an endvertex of one of two vertex-disjoint paths of lengths i, j,

$N_{i,j,k}$ $(k \geq j \geq i \geq 1)$ – the *generalized* (i,j,k)*-net*, i.e. the graph which is obtained by identifying each vertex of a triangle with an endvertex of one of three vertex-disjoint paths of lengths i, j, k.

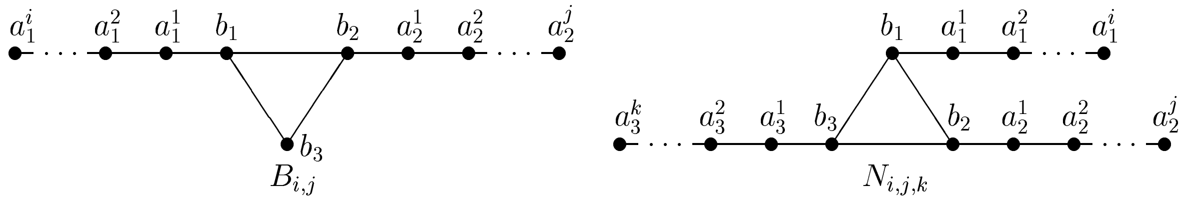


Figure 2

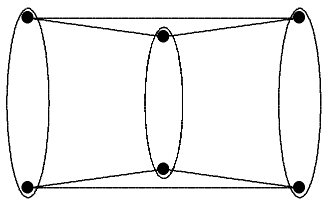
Thus, $B_{1,1} \simeq B$, $B_{1,2} \simeq W$ and $N_{1,1,1} \simeq N$. The graphs Z_i $(i \geq 1)$ can be defined analogously (see Figure 1). We further denote by H the *hourglass*, i.e. the graph consisting of two triangles with a common vertex.

A class $\mathcal{C}$ of graphs such that $\mathrm{cl}(G) \in \mathcal{C}$ for every $G \in \mathcal{C}$ is called a *stable class*. Clearly, the class of CA-free graphs is trivially stable if A is not claw-free or if A is not closed. The following theorem characterizes all connected closed claw-free graphs A for which the CA-free class is stable.

Theorem D [6]. *Let A be a closed connected claw-free graph. Then G being CA-free implies* $\mathrm{cl}(G)$ *is CA-free if and only if*

$$A \in \{H, C_3\} \cup \{P_i \mid i \geq 3\} \cup \{Z_i \mid i \geq 1\} \cup \{N_{i,j,k} \mid i, j, k \geq 1\}.$$

Making use of the special structure of closed claw-free graphs, the results on hamiltonicity in CP_i-free, CZ_i-free and CN-free graphs were extended in [5] and [6] to larger classes of CP_7-free, CZ_4-free and $CN_{1,2,2}N_{1,1,3}$-free graphs by characterizing the classes of nonhamiltonian exceptions. Specifically, the following was proved in [5] ($\mathcal{F}$ denotes the class of graphs shown in Figure 3, where the elliptical parts represent cliques of arbitrary order).



$\mathcal{F}$

Figure 3

Theorem E [5]. *If G is a 2-connected $CN_{1,1,2}$-free graph, then either G is hamiltonian or $G \in \mathcal{F}$.*

In the class of $CB_{i,j}$-free graphs, the following result was proved in [9].

Theorem F [9]. *Let $j \geq 2$ be an integer and let G be a 2-connected $CB_{2,j}$-free graph of diameter $d \geq \max\{7, 2j\}$. Then G is hamiltonian.*

In the main result of this paper we extend this result to the class of all 2-connected $CN_{1,2,j}$-free graphs.

2 Main result

It is easy to see that, for any $i, j, k \geq 1$, there are $CN_{i,j,k}$-free graphs of arbitrarily large diameter. A simple example can be obtained by taking $d+1$ vertex-disjoint cliques $K_0, K_1, \ldots, K_d$ (for sufficiently large d) and by adding all of the edges between consecutive cliques, namely $\{xy \mid x \in K_i, y \in K_{i+1}, i = 0, 1, \ldots, d-1\}$).

In the main result of this paper, Theorem 1, we show that nonhamiltonian 2-connected $CN_{1,2,j}$-free graphs must have small diameter.

Theorem 1. *Let $j \geq 2$ be an integer and let G be a 2-connected $CN_{1,2,j}$-free graph of diameter $d \geq \max\{7, 2j\}$. Then G is hamiltonian.*

Since every $B_{2,j}$-free or $N_{1,1,j}$-free graph is also $N_{1,2,j}$-free, Theorem 1 implies as immediate corollaries Theorem F and a corresponding result for $N_{1,1,j}$-free graphs. However, while it is shown in [9] that the assumptions of Theorem F are sharp, the lower bound on the diameter of G can be slightly improved in the case of $N_{1,1,j}$-free graphs.

Corollary 2. *Let $j \geq 2$ be an integer and let G be a 2-connected $CN_{1,1,j}$-free graph of diameter $d \geq \max\{4, 2j\}$. Then G is hamiltonian.*

The **proofs** of Theorem 1 and Corollary 2 are postponed to Section 3.

Remarks. 1. From [7] we know that every 2-connected $CN_{1,1,1}$-free graph is hamiltonian. The graph in Figure 4 indicates that there are 2-connected nonhamiltonian graphs of diameter $d = 6$ that are $CN_{1,2,j}$-free for any $j \geq 2$. The example in Figure 5 shows that there are 2-connected nonhamiltonian graphs which are $CN_{1,2,j}$-free and have diameter $d = 2j - 1$ for any $j \geq 3$. Hence the requirement $d \geq \max\{7, 2j\}$ in Theorem 1 is sharp.

Moreover, for any $j \geq 3$, the graph in Figure 6 is an example of a 2-connected nonhamiltonian $CN_{1,3,j}$-free graph of arbitrarily large diameter, and, similarly, the graph in Figure 7 is 2-connected, nonhamiltonian, $CN_{2,2,j}$-free and has also arbitrarily large diameter. Hence the assumption $CN_{1,2,j}$-free in Theorem 1 is also best possible.

It is easy to see that, in fact, each of the examples in Figures 4 – 7 yields an infinite family, since each of the vertical edges in the graphs of Figures 4, 5 (marked in the figure by K_i) and each of the edges incident with a vertex of degree 2 in the graphs of Figures 6, 7 can be blown up to a clique of arbitrary order.

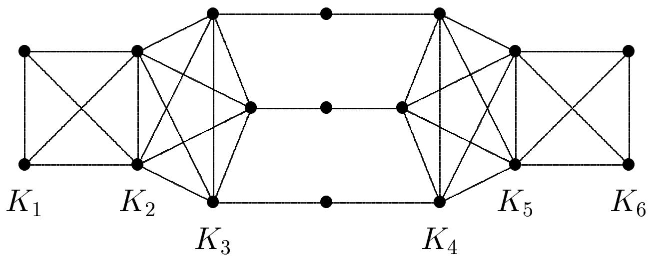


Figure 4

2. The graph in Figure 5 is also an example of a nonhamiltonian $CN_{1,1,j}$-free graph of diameter $d = 2j - 1$. Moreover, the graph in Figure 3 is a nonhamiltonian $CN_{1,1,2}$-free graph of diameter 3. Hence the assumption $d \geq \max\{4, 2j\}$ of Corollary 2 is also best possible.

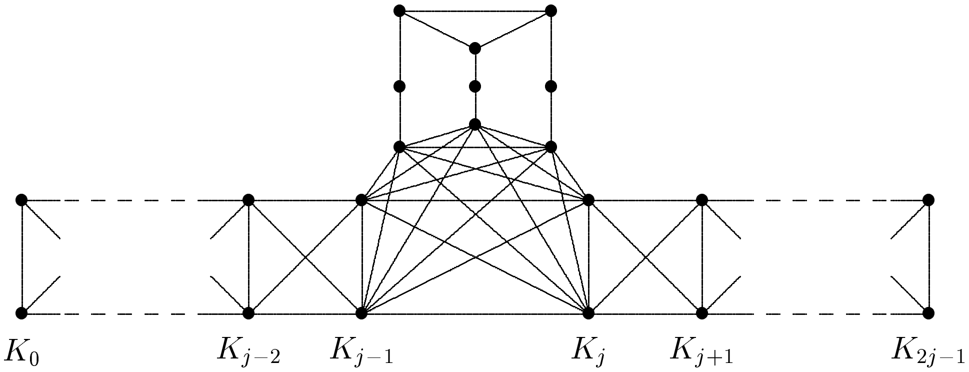


Figure 5

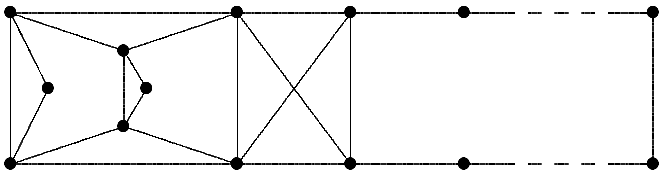


Figure 6

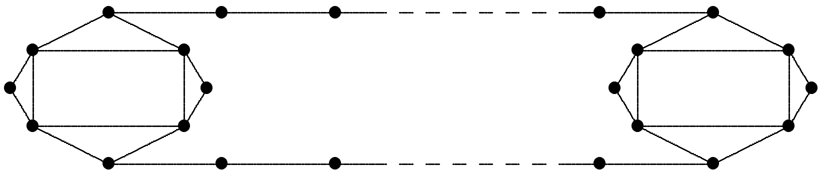


Figure 7

3 Proofs

Before we prove the main result of the paper, Theorem 1, we first make some preliminary observations on shortest paths and their neighborhoods. Their main idea can be found already in [7], however, for the sake of completeness, we include them here as well.

Let G be a claw-free graph, let $x, y \in V(G)$ and let $P: \ x = v_0v_1v_2 \dots v_k = y$, $k \geq 4$, be a shortest xy-path in G. Let $z \in V(G) \setminus V(P)$.

1. If $|N_P(z)| = 1$, then, since G is claw-free, z is adjacent to x or to y.
2. If $|N_P(z)| \geq 2$ and $\{v_i, v_j\} \subset N_P(z)$, then, since P is a shortest path, $|i - j| \leq 2$.
3. By (1) and (2), $|N_P(z)| \leq 3$ for every vertex $z \in V(G) \setminus V(P)$. Moreover, the vertices of $N_P(z)$ are consecutive on P.

This motivates the following notation:

$N_i := \{z \in V(G) \setminus V(P) | \ N_P(z) = \{v_{i-1}, v_i, v_{i+1}\}\}$ for $1 \leq i \leq k-1$,

$M_i := \{z \in V(G) \setminus V(P) | \ N_P(z) = \{v_{i-1}, v_i\}\}$ for $1 \leq i \leq k$,

$$M_0 := \{z \in V(G) \setminus V(P) |\ N_P(z) = \{v_0\}\},$$

$$M_{k+1} := \{z \in V(G) \setminus V(P) |\ N_P(z) = \{v_k\}\}.$$

Thus, by (1), (2) and (3), $N(P) \cup V(P) = (\bigcup_{i=1}^{k} N_i) \cup (\bigcup_{i=0}^{k+1} M_i) \cup V(P)$.

We further denote $S = V(P) \cup N(P)$, $R = V(G) \setminus S$, $M_i^* = N_R(M_i)$, $1 \le i \le k$, and $N_i^* = N_R(N_i)$, $1 \le i \le k-1$.

The sets M_i, N_i, M_i^*, N_i^* have the following properties.

Lemma 3. *Let G be a claw-free graph, $x, y \in V(G)$, and let $P:\ x = v_0v_1v_2\dots v_k = y$ ($k \ge 3$) be a shortest x,y-path in G. Then*

(*i*) *$\langle N_i \cup \{v_i\}\rangle_G$ is complete for $1 \le i \le k-1$ and $\langle M_i \cup \{v_{i-1}, v_i\}\rangle_G$ is complete for $1 \le i \le k$,*

(*ii*) *$\langle N_{i-1} \cup M_i \cup \{v_{i-1}, v_i\}\rangle_G$ and $\langle M_i \cup N_i \cup \{v_{i-1}, v_i\}\rangle_G$ are complete for $2 \le i \le k-1$,*

(*iii*) *$N_i^* = \emptyset$ for $1 \le i \le k-1$.*

Proof. (*i*) If some N_i or M_i is not complete, then some v_j, $j \in \{i-1, i, i+1\}$, is a center of an induced claw, a contradiction.

(*ii*) If $x \in N_{i-1}$ and $y \in M_i$ are nonadjacent, then $\langle\{v_i, x, y, v_{i+1}\}\rangle_G$ is an induced claw. Hence $\langle N_{i-1} \cup M_i\rangle_G$ (and similarly also $\langle M_i \cup N_i\rangle_G$) is complete; v_{i-1} and v_i are adjacent to all its vertices.

(*iii*) If $x \in N_i^*$, then x has a neighbor y in N_i and $\langle\{y, x, v_{i-1}, v_{i+1}\}\rangle_G$ is a claw. ■

If, moreover, G is net-free, then we know more about its structure.

Lemma 4. *Let $j \ge 2$, let G be a $CN_{1,2,j}$-free graph, let $x, y \in V(G)$ and let $P:\ x = v_0v_1v_2\dots v_k = y$ be a shortest xy-path of length $k \ge \max\{7, 2j\}$ in G. Then*

(*i*) *$M_i^* = \emptyset$ for $3 \le i \le k-2$,*

(*ii*) *for every vertex $z \in R$ we have $N_P(z) = \emptyset$ and $N_S(z) \subseteq M_0 \cup M_1 \cup M_2 \cup M_{k-1} \cup M_k \cup M_{k+1}$.*

Proof. (*i*) Suppose $x \in M_i^*$ for some i, $3 \le i \le k-2$, and let y be a neighbor of x in M_i. Then for $i \le \lfloor k/2\rfloor$ we have $\langle\{v_{i-3}, v_{i-2}, v_{i-1}, y, x, v_i, \dots, v_{i+j}\}\rangle_G \simeq N_{1,2,j}$. The case $i > \lfloor k/2\rfloor$ is symmetric.

(*ii*) $N_P(z) = \emptyset$ follows from the definition of R; the rest follows from (3) of Lemma 3 and from (1) of Lemma 4. ■

Proof of Theorem 1. Let G be a 2-connected $CN_{1,2,j}$-free graph of diameter $d \ge \max\{7, 2j\}$, $j \ge 2$, and let $P:\ v_0v_1v_2\dots v_d$ be a diameter path in G. Let M_i, N_i, M_i^*, S, R be as above. Set $c = \lfloor d/2\rfloor$.

We distinguish two cases.

<u>Case 1:</u> $M_c \cup N_c \cup \{v_c\}$ is not a cutset of G.

Let $H = G - \{M_c \cup N_c \cup \{v_c\}\}$ and let $P' : v_d v_{d+1} v_{d+2} \dots v_{d+\ell-1} v_{d+\ell} = v_0$ be a shortest $v_d v_0$-path in H. Since P is a diameter path, $\ell \geq d$. Since H is $CN_{1,2,j}$-free and P' is a shortest path in H, we can define analogously the sets M_i, N_i, M_i^* for $i = d+1, \dots d+\ell$. Specifically, by (1) of Lemma 4 we have $M_i^* = \emptyset$ for $d+3 \leq i \leq d+\ell-2$.

Set $H' = G - \{M_2 \cup M_{d+\ell-2} \cup N_2 \cup N_{d+\ell-2} \cup \{v_2, v_{d+\ell-2}\}\}$. By virtue of (1) of Lemma 4, there is a shortest $v_3 v_{d+\ell-3}$-path in H' containing all vertices v_i for $i \in \{3, \dots, d-2\} \cup \{d+2, \dots, d+\ell-3\}$. Let $P_d : v_3 v_4 \dots v_{d-2} P^d v_{d+2} \dots v_{d+\ell-3}$ be such a path (where P^d is a $v_{d-2} v_{d+2}$-path). If $v_{d-2} v_{d+2} \in E(G)$, then $v_{d+2} \in M_{d-1}$, implying $\langle \{v_{d+4}, v_{d+3}, v_{d+2}, v_{d-1}, v_d, v_{d-2}, \dots, v_{d-(j+2)}\} \rangle_G \simeq N_{1,2,j}$, and if $v_{d-2} v_{d+2}$ are at distance 2 with a common neighbor x, then we have similarly $x \in M_{d-1}$ and $\langle \{v_{d+3}, v_{d+2}, x, v_{d-1}, v_d, v_{d-2}, \dots, v_{d-(j+2)}\} \rangle_G \simeq N_{1,2,j}$. Hence P^d is of length at least 3. Then the length of P_d is at least $(d-5) + 3 + (\ell - 5) = d + \ell - 7 \geq \max\{7, 2j\}$ (since $\ell \geq d$ and $d \geq \max\{7, 2j\}$). Hence $N(P^d)$ has the structure described in Lemma 3 and Lemma 4.

Let, symmetrically, $P_0 : v_{d+3} v_{d+4} \dots v_{d+\ell-2} P^0 v_2 \dots v_{d-3}$ be a shortest $v_{d+3} v_{d-3}$-path in the graph $H' = G - (M_{d-2} \cup M_{d+2} \cup N_{d-2} \cup N_{d+2} \cup \{v_{d-2}, v_{d+2}\})$ (where again P^0 is a $v_{d+\ell-2} v_2$-path of length at least 1). Define the cycle C by $C : v_2 v_3 \dots v_{d-2} P^d v_{d+2} v_{d+3} \dots v_{d+\ell-2} P^0 v_2$. Then C is a (chordless) cycle of length $\ell' \geq d - 5 + 3 + \ell - 5 + 3 \geq 2d - 4 > d$. Relabel the vertices of P, P' such that $C : v_1, v_2 \dots v_{\ell'} v_1$, and define the sets M_i, N_i, M_i^*, N_i^* accordingly (indices modulo ℓ'). Then $M_i^* = N_i^* = \emptyset$ for all $i = 1, \dots, \ell'$, implying that $(\cup_{i=1}^{\ell'} (M_i \cup N_i)) \cup V(C) = V(G)$, i.e., $V(G) = V(C) \cup N(C)$. Since clearly v_i is eligible in G if and only if $N_i \neq \emptyset$ or $E(M_i, M_{i+1}) \neq \emptyset$, we conclude that $\mathrm{cl}(G)$ is either complete (if $N_i = \emptyset$ or $E(M_i, M_{i+1}) = \emptyset$ for at most three $i = 1, \dots, \ell'$), or otherwise $\mathrm{cl}(G)$ consists of k cliques $K_1, \dots, K_k$ ($4 \leq k \leq \ell'$) such that $|V(K_i) \cap V(K_j)| = 1$ for $|i - j| \in \{1, k-1\}$ and $V(K_i) \cap V(K_j) = \emptyset$ for $i \neq j$ otherwise. In both cases, the hamiltonicity of $\mathrm{cl}(G)$ (and hence also of G) is apparent.

Case 2: $M_c \cup N_c \cup \{v_c\}$ is a cutset of G.

Then, by (1) of Lemma 4 and (2) of Lemma 3, each of $\langle M_i \cup N_i \cup \{v_i\} \rangle_G$ is a clique cutset of G for $i = 2, \dots, d-2$.

First observe that $M_0 \subset N(M_2)$ and $M_1^* \subset M_2^*$, for otherwise there is a vertex $x \in M_0 \cup M_1^*$ at distance $d+1$ from v_d (since every $x v_d$-path passes through $M_2 \cup N_2 \cup \{v_2\}$). Symmetrically, $M_{d+1} \subset N(M_{d-1})$ and $M_d^* \subset M_{d-1}^*$. Set

$$G_1 = \langle \{v_0, v_1, v_2\} \cup N_1 \cup N_2 \cup M_0 \cup M_1 \cup M_2 \cup M_2^* \rangle_G,$$

$$G_2 = \langle \{v_3, \dots, v_{d-3}\} \cup N_3 \cup \dots N_{d-3} \cup M_3 \cup \dots M_{d-2} \rangle_G,$$

$$G_3 = \langle \{v_{d-2}, v_{d-1}, v_d\} \cup N_{d-2} \cup N_{d-1} \cup M_{d-1} \cup M_d \cup M_{d+1} \cup M_{d-1}^* \rangle_G.$$

Since G is 2-connected, for any $i = 2, \dots, d-2$ we have either $N_i \neq \emptyset$, or $M_i \neq \emptyset$, $M_{i+1} \neq \emptyset$ and $E(M_i, M_{i+1}) \neq \emptyset$. This implies that v_i is eligible in G for all i, $2 \leq i \leq d-2$, and hence $V(G_2)$ induces a clique in $\mathrm{cl}(G)$.

If both $V(G_1)$ and $V(G_3)$ induce a clique in $\mathrm{cl}(G)$, then $\mathrm{cl}(G)$ is complete and we are done. Hence, by symmetry, suppose that $\langle V(G_1) \rangle_{\mathrm{cl}(G)}$ is not complete. This immediately implies that v_1 cannot be eligible in G (recall that $M_0 \subset N(M_2)$ and $M_1^* \subset M_2^*$), hence $N_1 = \emptyset$ and $E(M_1, M_2) = \emptyset$. Since G is 2-connected, v_1 cannot be a cutvertex, implying $M_0 \neq \emptyset$ or $M_1^* \neq \emptyset$. Then it is straightforward to check that in all possible cases (according to which of M_0, M_1^* is nonempty), for any $v_2' \in N_2 \cup M_2$ there is a v_2v_2'-path P_1 such that $V(P_1) = V(G_1)$ (note that $\langle M_0 \rangle_G$ and all $\langle N_R(v) \rangle_G$, $v \in M_2$, are complete since G is claw-free). By Theorem B (iv), G is hamiltonian. ■

Proof of Corollary 2. Since $N_{1,1,j}$-free implies $N_{1,2,j}$-free, we immediately have the result for $d \geq \max\{7, 2j\}$. If G is $N_{1,1,j}$-free, then in Lemma 4 (i) we moreover get $M_i^* = \emptyset$ for $2 \leq i \leq d-1$, which, reconsidering the proof of Theorem 1, yields the result for $d \geq \max\{5, 2j\}$. Finally, every $CN_{1,1,2}$-free graph of diameter at least 4 must be hamiltonian by Theorem E. ■

References

[1] Bedrossian, P.: Forbidden subgraph and minimum degree conditions for hamiltonicity. Thesis, Memphis State University, U.S.A., 1991

[2] Bondy, J.A.; Murty, U.S.R.: Graph theory with applications. Macmillan, London and Elsevier, New York, 1976.

[3] Broersma, H.J.; Veldman, H.J.: Restrictions on induced subgraphs ensuring hamiltonicity or pancyclicity of $K_{1,3}$-free graphs. Contemporary methods in Graph Theory (R. Bodendiek), BI-Wiss.-Verl., Mannheim-Wien-Zürich, 1990, 181-194.

[4] Brousek, J.: Minimal 2-connected non-hamiltonian claw-free graphs. Discrete Math. 191 (1998), 57-64.

[5] Brousek, J., Favaron, O., Ryjáček, Z.: Forbidden subgraphs, hamiltonicity and closure in claw-free graphs. Discrete Math. 196 (1999), 29-50.

[6] Brousek, J., Ryjáček, Z., Schiermeyer, I.: Forbidden subgraphs, stability and hamiltonicity. Discrete Math. 197/198 (1999), 143-155.

[7] Duffus, D.; Gould, R.J.; Jacobson, M.S.: Forbidden subgraphs and the hamiltonian theme. The theory and applications of graphs. (Kalamazoo, Mich. 1980), Wiley, New York, 1981 297-316.

[8] Faudree, R.J.; Gould, R.J.: Characterizing forbidden pairs for hamiltonian properties. Discrete Math. 173(1997), 45-60.

[9] Faudree, R.J.; Ryjáček, Z.; Schiermeyer, I.: Claw-free and generalized bull-free graphs of large diameter are hamiltonian. Tatra Mountains Math. Publications 18 (1999), 105-113.

[10] Goodman, S.; Hedetniemi, S.: Sufficient conditions for a graph to be hamiltonian. J. Combin. Theory Ser. B 16(1974) 175-180.

[11] Gould, R.J.; Jacobson, M.S.: Forbidden subgraphs and hamiltonian properties of graphs. Discrete Math. 42(1982), 189-196.

[12] Ryjáček, Z.: On a closure concept in claw-free graphs. J. Combin. Theory Ser. B 70(1997), 217-224.